CHAPTER 7
SEEPAGE TOWARD WELLS

7-1. Use of Wells. Wells are used in a variety of ways to control seepage. They may be placed-on the landward side of water retention structures to reduce pressure at the lower boundary of impervious strata. Wells are also used to maintain dry conditions in excavations during construction. In addition to seepage control, well pumping tests serve as an accurate means of field determination of permeability (see Chapter 2).

7-2. Analysis of Well Problems. The graphical flow net technique described in Chapter 4 or the approximate methods described in Appendix B can be used in the analysis of well problems. However, formulas obtained from analytical solutions to well problems are the most common methods of analysis.

a. Flow Nets. An example of a flow net for a simple flow problem is shown in figure 7-1. The flow between flow lines is given by (Taylor 1948)

$$\Delta Q = 2\pi k \Delta h \frac{rb}{\ell} \tag{7-1}$$

where

k = permeability (L/T)

Δh = total head loss between equipotential lines (L)

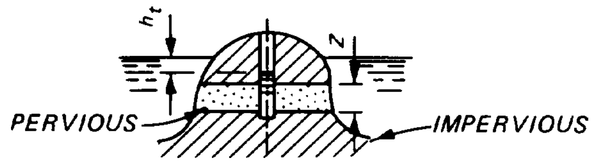
r = distance from well (L)

b = dimension of element in Z direction (L)

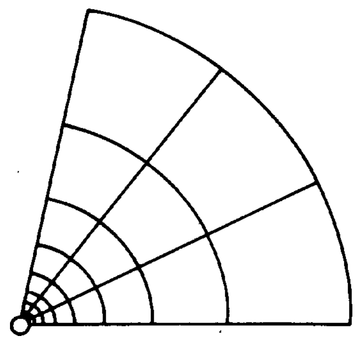
ℓ = dimension of element in r direction (L)

As for a plan flow net, ΔQ and Δh must be the same for all elements within the net. Thus rb/ℓ is a constant. When drawn in plan view (figure 7-1b) the flow net consists of square elements as in the plane case described in Chapter 4. When drawn in profile (figure 7-1c) the elements' aspect ratios (b/R) are proportional to the radial distance r and are therefore not squares. Thus, graphical construction of flow nets for radial flow problems is generally not practical except for cases where the water bearing has a constant thickness and only the plan view of the net is required.

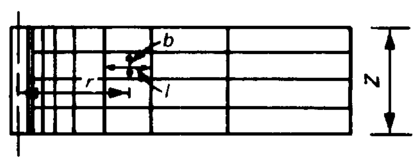
b. Approximate Solutions. The numerical and analog methods described in Appendix C can be used for problems involving complicated boundary conditions. Electrical analog methods are especially advantageous as most complicated well problems cannot be idealized in two dimensions.



a. HORIZONTAL FLOW TO WELL



b. PLAN VIEW OF FLOWNET



c. PROFILE VIEW OF FLOWNET

Figure 7-1. Flowout of simple radial flow problem (courtesy of John Wiley and Sons[268])

c. Analytical Formulas. The analysis of flow to a single well can often be solved by analytical methods. Also, the analysis of flow to multiple wells and many problems involving complicated boundary conditions can be solved by superposition of solutions for single well problems. Analytical solutions can be obtained for nonsteady flow problems.

7-3. Basic Well Equations for Steady State Flow. Steady flow conditions exist when the well flow rate and piezometric surface do not change with time. If the regional piezometric surface does not fluctuate, steady state conditions are achieved by pumping from a well at a constant rate for a long time period. Design of wells for seepage control are often based on computations assuming steady state conditions.

a. Artesian Conditions. When significant flow to a well is confined to a single saturated stratum, the problem can be idealized as shown in figure 7-2a. An artesian condition exists when the height h of the piezometric surface lies above the top of the water bearing unit b . If the properties of the soil are constant in all directions from the well, the discharge Q from the well must be equal to the flow through a cylinder defined by the radius r , height b , and differential thickness dr . Thus from Darcy's law (equation 3-3), the flow can be written as

$$Q = k \frac{dh}{dr} 2\pi rb \tag{7-2}$$

where

Q = constant discharge from well (L^3/T)

k = coefficient of permeability -(L/T)

$\frac{dh}{dr}$ = hydraulic gradient along radius (L/L)

r = radius of cylinder (L)

B = thickness of aquifer (L)

Upon integrating equation 7-2, the relationship between r and h is found.

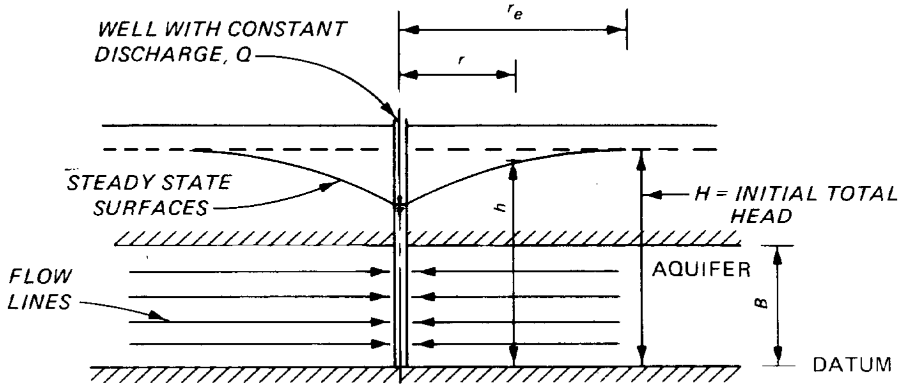
$$h = \frac{Q}{2\pi kB} \ln r + \text{constant} \tag{7-3}$$

The constant can be determined by specifying that at the radius r_e , the total head h is equal to a known head H , the total head that existed before starting discharge from the well. That is,

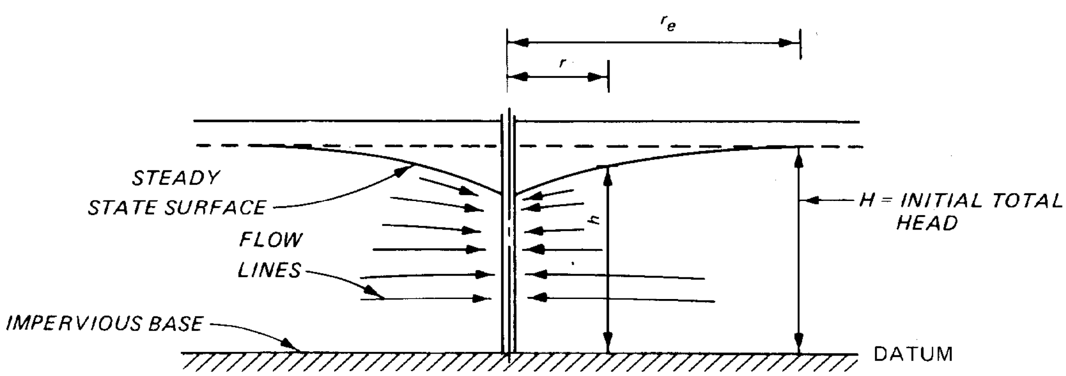
$$h = H \text{ for } r = r_e \tag{7-4}$$

Inserting equation 7-4 into equation 7-3, the constant term is found to be:

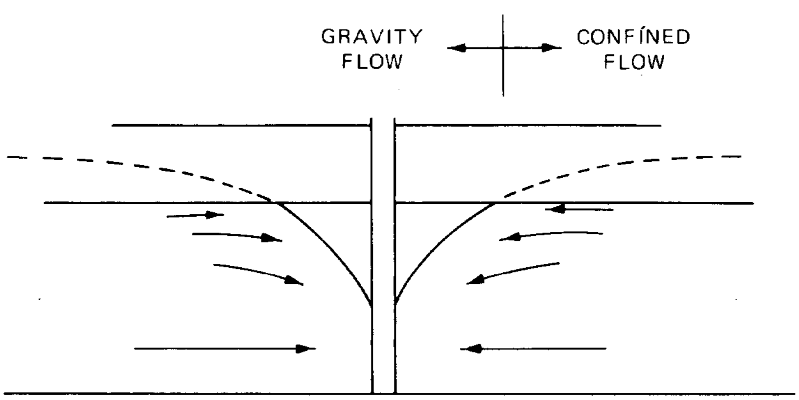
$$\text{constant} = H - \frac{Q}{2\pi kB} \ln r_e$$



a. HORIZONTAL AQUIFER CONFINED BETWEEN IMPERVIOUS STRATA (ARTESIAN FLOW)



b. HORIZONTAL UNCONFINED AQUIFER (GRAVITY FLOW)



c. COMBINED AND CONFINED AND GRAVITY FLOW

Figure 7-2. Radial flow to horizontal aquifers (courtesy of John Wiley and Sons[164])

By substituting the constant term into equation 7-3 and combining logarithmic terms, the well equation for confined flow is obtained.

$$H - h = \frac{Q}{2\pi kB} \ln \frac{r_e}{r} \qquad (7\text{-}5)$$

The distance r_e is often defined as the radius beyond which the well has no influence or radius of influence.

b. Gravity Flow Conditions. Flow to a well under gravity (figure 7-2b) differs from the confined flow problem in the important aspect that the height B of the differential cylinder is equal to the variable h . Thus, equation 7-2 must be written as:

$$Q = k \frac{dh}{dr} 2\pi rh \qquad (7\text{-}6)$$

which upon integration and substitution of boundary condition gives

$$h^2 = \frac{Q}{\pi k} \ln r + \text{constant} \qquad (7\text{-}7)$$

The constant term can be evaluated from the condition at the radius of influence r_e as was done in equations 7-4 and 7-5. The constant term is given by:

$$\text{constant} = H^2 - \frac{Q}{\pi k} \ln r_e$$

which when substituted back into equation 7-7 gives the well equation for gravity flow

$$H^2 - h^2 = \frac{Q}{\pi k} \ln \frac{r_e}{r}$$

Development of equation 7-6 is based on the Dupuit assumption (Chapter 4) which limits the applicability of equation 7-7 to those cases where the slope of the piezometric surface is small (less than 5 percent). The error is greatest in the vicinity of the well.

c. Combined Artesian and Gravity Flow. When drawdown of the potentiometric surface becomes large near the well, combined gravity and confined conditions can occur (figure 7-2c).

d. Flow to Well Groups (Method of Superposition). The piezometric surface h caused by discharge from a group of wells can be determined by superimposing the solution for the individual wells given by either equation 7-3 or 7-6. For multiple wells, flow cannot be idealized by concentric cylinders and the problem must be stated in terms of the plan coordinates x and y (figure 7-3b). By noting that for a well located at

(x_i, y_i), $r_i^2 = (x - x_i)^2 + (y - y_i)^2$ a general well equation can be written as

$$\phi_i = q_i \ln (r_i) + C_i \tag{7-8}$$

where

ϕ_i = potential required at point (x,y) to sustain a discharge Q_i from a well at (x_i, y_i)

= h_i (confined flow)

= h_i^2 = (unconfined or gravity flow)

q_i = intensity factor

= $Q_i/4\pi kB)$ (confined flow)

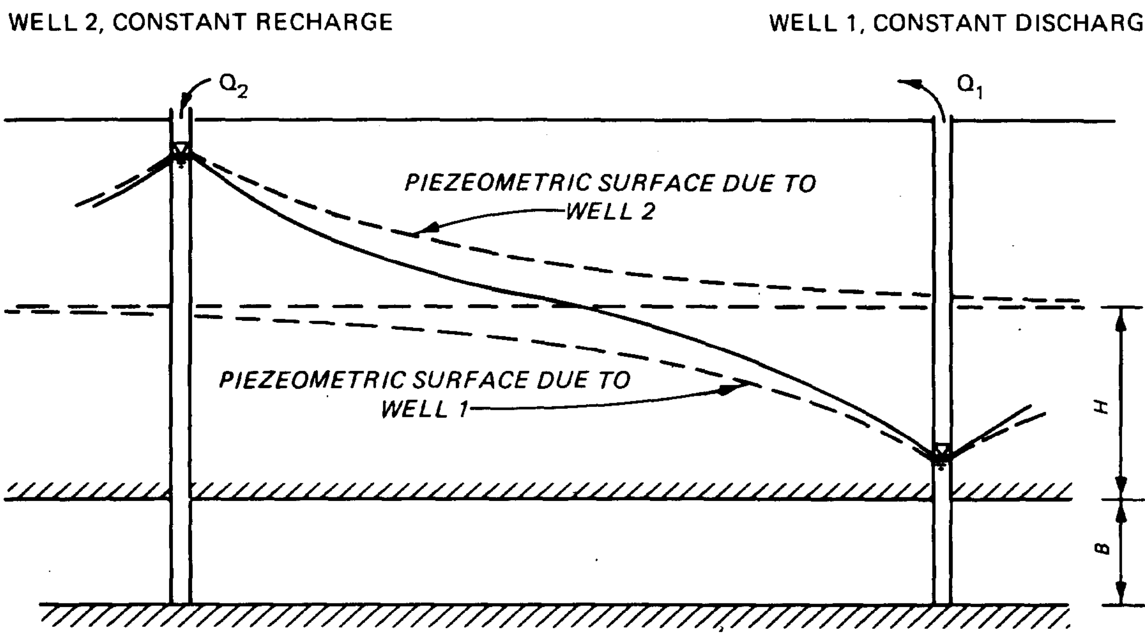
= $Q_i/(\pi k)$ (unconfined or gravity flow)

C_i = constant

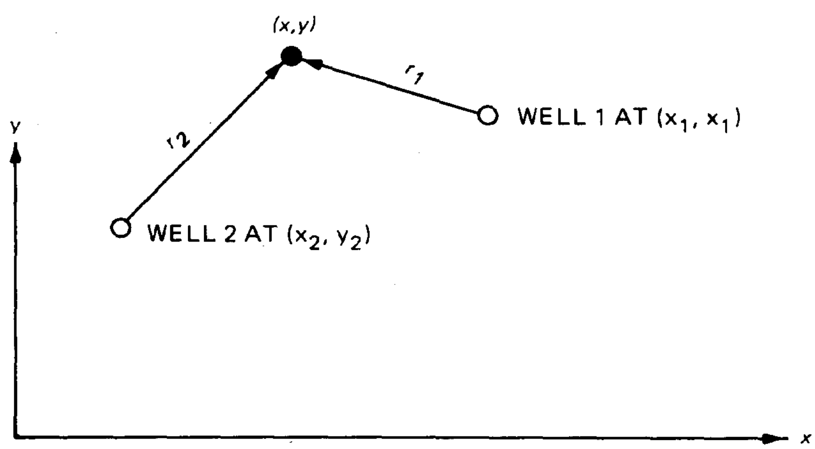
The head distribution $\phi(x,y)$ can be determined by summing the individual ϕ_i . As the sum of the constants C_i is a constant, the multiple well equation can be written as

$$\phi(x,y) = \sum_{i=1}^{n} q_i \ln (r_i) + \text{constant} \tag{7-9}$$

where n is the number of wells. The constant is determined from a known value of ϕ at a specified location. For example, the superposition formula



a. FLOW BETWEEN TWO WELLS



b. COORDINATE SYSTEM FOR FLOW TO MULTIPLE WELLS

Figure 7-3. Flow to multiple wells (adapted from John Wiley and Sons[164]) for the wells shown in figure 7-3 would be

$$\emptyset(x,y) = h(x,y) = h_1(x,y) + h_2(x,y)$$

$$= \frac{1}{4\pi kB}\left[Q_1 \ \ell n[x - x_1)^2 + (y - y_1)^2\right]$$

$$- Q_2 \ \ell n\left[x - x_2)^2 + (y - y_2)^2\right] + \text{constant}$$

At a distance r_e from both wells h(x,y) = H . The constant term is found to be

$$\text{constant} = \frac{\ell n \ r_e}{4\pi kB}(Q_1 - Q_2)$$

Substituting the above equation into equation 7-9, the well formula for two wells becomes

$$H - h(x,y) = \frac{1}{4\pi kB}\left[Q_1 \ \ell n \frac{r_1^2}{r_e^2} - Q_2 \ \ell n \frac{r_2^2}{r_e^2}\right]$$

e. Hydrologic Boundaries (Image Well Method). When there are hydrologic boundaries within the radius of influence of the well, equations 7-3 and 7-7 are no longer valid. Examples of boundaries are:

(1) A stream or river which can be idealized as a line source of equal potential.

(2) A rock bluff line at the edge of an alluvial fill valley which can be idealized as an impervious boundary.

The superposition of solutions (equation 7-9) can be used to analyze the flow near a boundary by introducing an artificial device called an image well. An image well is identical to the actual well and located symmetrically on the opposite side of the boundary. The superimposed effect of the real and image well for an infinite well is identical to the influence of the real well and boundary. If the real well is a pumping well then a recharging image well is used to represent boundaries such as rivers and a pumping image well is used to represent an impervious barrier. For either case, the absolute value of the flow Q for the image well is equal to that of the real well. For example, the head distribution created by a discharging well in the vicinity of a river is identical to that created by the combined influence of a recharge and discharge well (see figure 7-4). The head distribution created by the discharge well in an infinite confined aquifer is given by

$$h_R = \frac{Q}{2\pi kB} \ell n\ r_R + \text{constant} \tag{7-10a}$$

and by the image recharge well in the infinite aquifer

$$h_I = \frac{-Q}{2\pi kB} \ell n\ r_I + \text{constant} \tag{7-10b}$$

By superposition, the head distribution for the true actual problem is

$$h = h_R + h_I$$

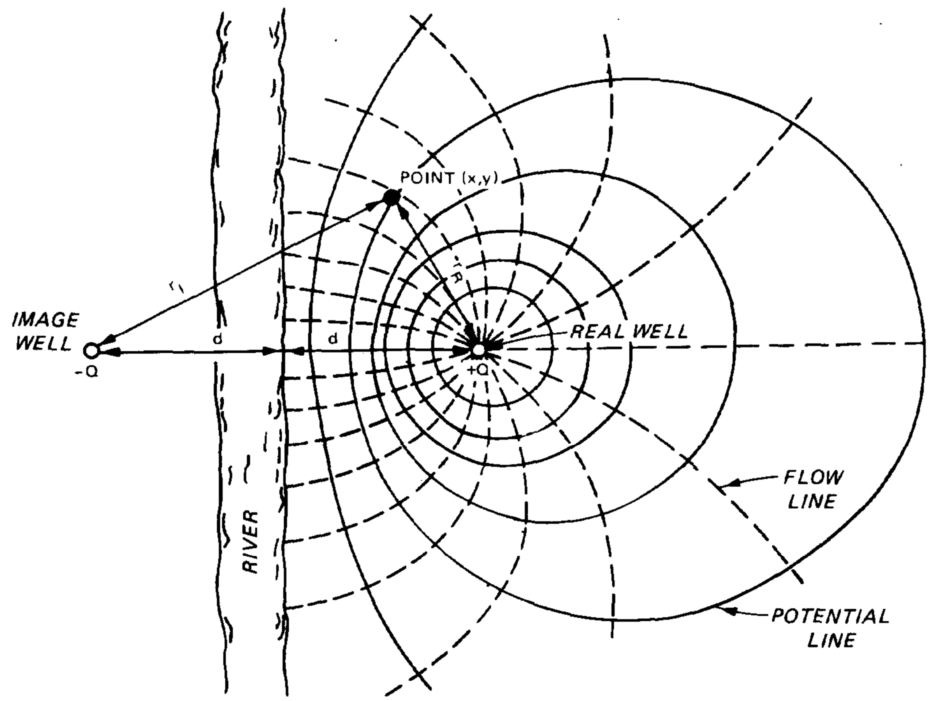
$$= \frac{Q}{2\pi kB} \ell n \frac{r_R}{r_I} + \text{constant} \tag{7-11}$$

Note that at the river $r_I = r_R$ and $\ell n \frac{r_I}{r_R} = 0$ Thus, constant = H , the head at the river. Substituting the constant term into equation 7-11, the formula for a single well near a recharge boundary is

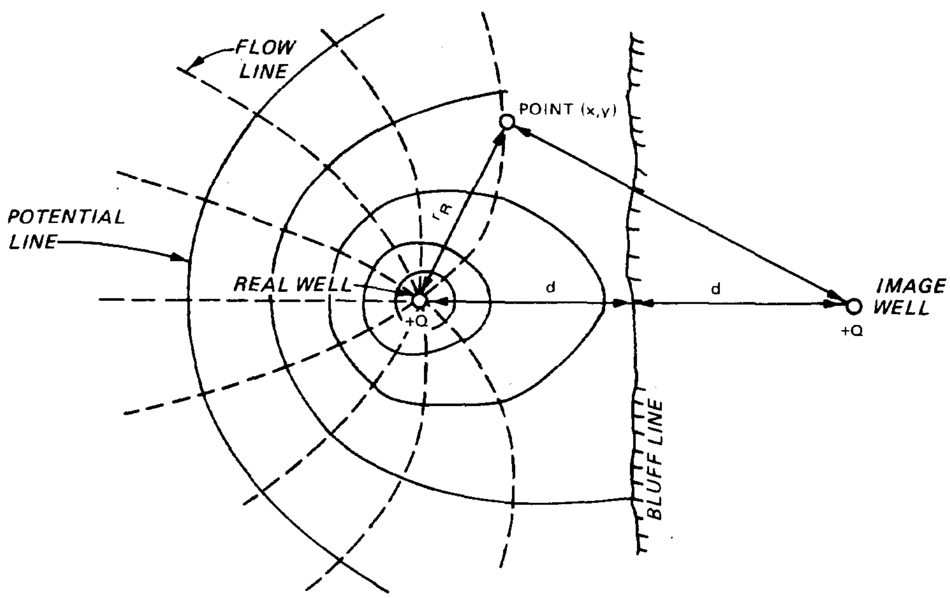
$$H - h = \frac{Q}{2\pi kB} \ell n \frac{r_I}{r_R}$$

To describe the head distribution for confined flow near an impervious boundary an image discharge well is used (figure 7-4b), By the procedure used above, h would be obtained as

$$h = \frac{Q}{2\pi kB} \ell n\ r_R r_I + \text{constant} \tag{7-12}$$



a. IMAGE WELL ANALYSIS OF DISCHARGE WELL NEAR RECHARGE BOUNDARY (RIVER)



b. IMAGE WELL ANALYSIS OF DISCHARGE WELL NEAR IMPERMEABLE BOUNDARY (ROCK BLUFF)

Figure 7-4. Application of image well method for analysis of flow near boundaries (courtesy of Illinois State Water Survey[287])

The head at the impervious boundary is unknown, thus additional information is needed to determine the constant. Note that when r_R and r_I are both equal to the radius of influence that h = H . Thus

$$h = \frac{Q}{2\pi kB} \ln\left(\frac{r_R\ r_I}{r_e^2}\right) + H \tag{7-13}$$

The image well method can also be applied to problems involving multiple boundaries. For example, a common geologic situation involving multiple boundaries would be a discharge well pumping from an alluvial terrace located between a river and rock bluff (figure 7-5). In this case, the image well for the river would have a second image well with respect to the rock bluff, which in turn would have an image with respect to the river and so on. A similar progression of image wells would be needed for the impermeable barrier. Eventually, the location of each added-image well extends beyond its radius of influence r_e from the pumping well and has no practical influence in the solution.

7-4. Special Conditions. Although the simple well formula (equation 7-8) is often used to analyze flow problems, it describes a relatively idealized condition that is found rarely in practice. It is generally desirable to consider the effects of partial penetration of wells, sloping aquifer, and stratification of water bearing units in the analysis.

a. Partially Penetrating Wells. In deriving equations 7-3 and 7-7 it is assumed that the flow lines are horizontal at the entrance of the well. This assumption is valid only if the well completely penetrates the water bearing strata. An approximate solution for flow to a well partially penetrating a confined aquifer was developed by Muskat (1946). The head can be computed from

$$h = C_1 - C_2\beta \tag{7-14}$$

where C_1 and C_2 are constants to be determined from boundary conditions and β is a function of the radius from the well (Warriner and Banks 1977). The expression for β given by Muskat (1946) was based on simplifying assumptions. Duncan (1963) and Banks (1965) assessed its validity from electrical analogy model studies and developed a more accurate expression for β . The alternative empirically determined relationship for β developed by Duncan (1963) is given in figure 7-6. The constants C_1 and C_2 are determined

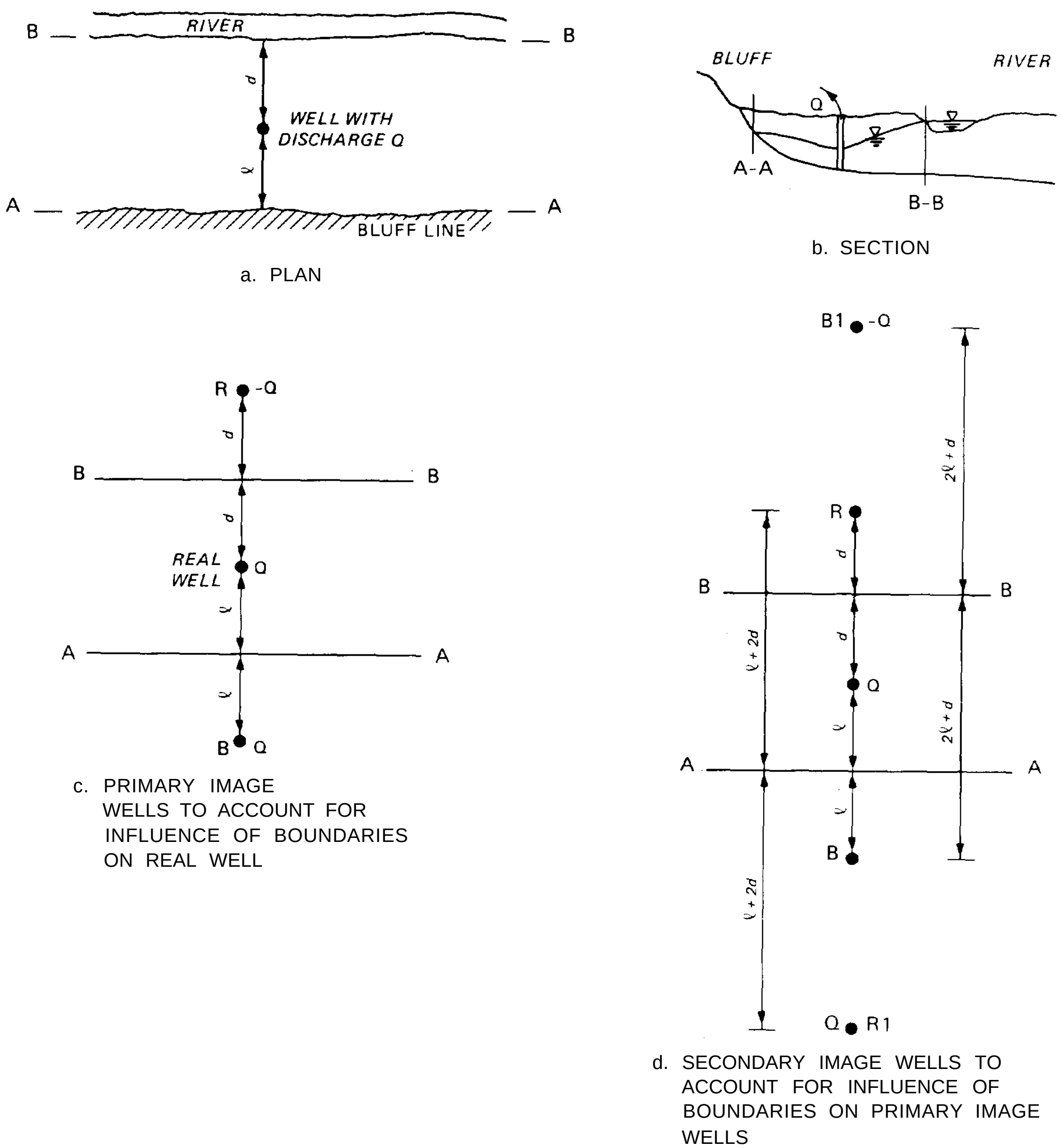


Figure 7-5. Multiple image wells for a two-boundary problem

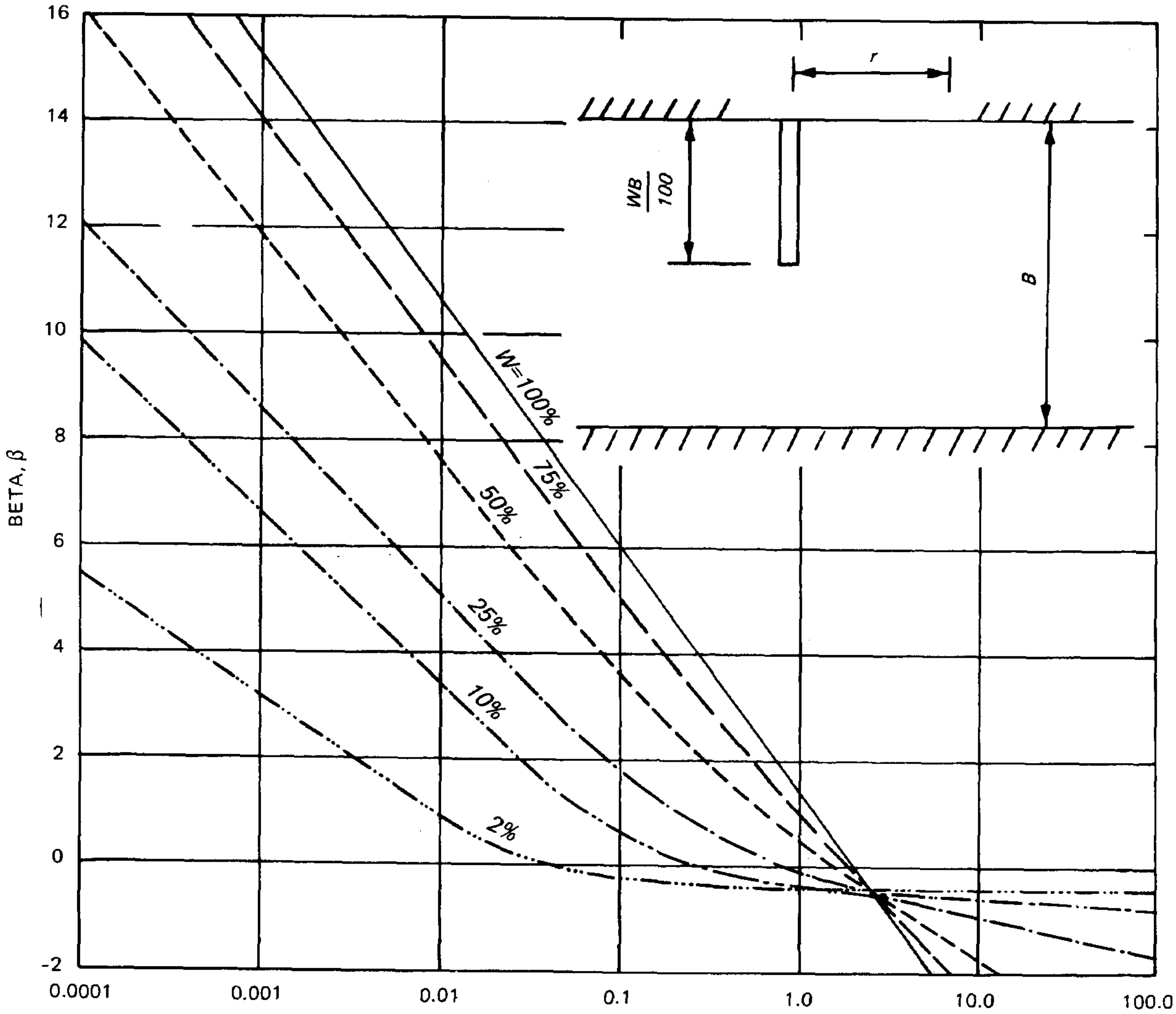


Figure 7-6. Beta function curve (from Warriner and Banks[124])

from the boundary conditions at the well and at the radius of influence as

$$C_1 = h_w + C_2\beta_w \tag{7-15}$$

$$C_2 = \frac{H - h_w}{\beta_w - \beta_e}$$

where

h_w = total head at well (L)

β_w = value of β at well radius r_w (dimensionless)

H = total head at radius of influence r_e (L)

β_e = value of β at radius of influence r_e (dimensionless)

The well discharge can be determined by using an empirically determined shape factor $\mathcal{S}$

$$Q = K(H - h_w)\mathcal{S}B \tag{7-16}$$

with

$$\mathcal{S} = \frac{2\pi}{\ell n \frac{r_e}{4B}} \frac{\beta_{4d} - \beta_e}{\beta_w - \beta_e}$$

where

β_{4d} = value of β at $r = 4B$

B = aquifer thickness

b. Flows to Groups of Partially Penetrating Wells. An empirical method developed by Warriner and Banks (1977) provides a means to modify the relationship obtained by superimposing solutions for individual fully penetrating wells for the effects of partial penetration. First, the head at each well is computed from the assumption that they fully penetrate the aquifer:

$$h_j = c + \frac{1}{2\pi kB} \sum_{i=1}^{N} Q_i \, \ell n \frac{r_{ij}}{a} \tag{7-17}$$

with $r_{jj} = r_{wj}$

where

h_j = head at well j (L)

c = constant of integration (L)

Q_i = discharge from well i (L^3/T)

k = coefficient of permeability (L/T)

B = aquifer thickness (L)

a = constant (L)

r_{ij} = distance between well i and well j (L)

r_{wj} = radius of well j (L)

N = number of wells in group

In addition, the head at a point on the source boundary is given by:

$$H = c + \frac{1}{2\pi kB} \sum^{N} Q_i \ \ell n \frac{r_{is}}{a} \qquad (7\text{-}18)$$

where H is the head at the source and r_{is} is the distance between well i and the source. The drawdown at each well is computed from combining equations 7-17 and 7-18

$$\Delta hj = H - hj = \sum_{i=1}^{N} \frac{Q_i}{2\pi kB} \ \ell n \frac{r_{js}}{r_{ij}} \qquad (7\text{-}19)$$

with $r_{jj} = r_{wj}$.

Equation 7-19 gives the drawdown for each well within a group of fully penetrating wells. The values of Q_i required to cause the drawdown Δh_j can be determined by solving the system of N equations (7-19) for the N unknowns Q_i . As for the single well, a shape factor can be defined as:

$$\mathcal{S}_i = \frac{Q_i}{k\Delta h_i B} \qquad (7\text{-}20)$$

where $\$_i$ is the shape factor for each well within a group. This shape factor can be corrected to account for partial penetration by

$$\$'_i = \frac{\ln \frac{r_{is}}{r_{iw}}}{\ln \frac{r_{is}}{4B}} \left(\frac{\beta_{4d} - \beta_e}{\beta_w - \beta_e} \right) \tag{7-21}$$

By replacing $\$_i$ in equation 7-16 with $\$'_i$ the flow from the well group is given as

$$Q_{total} = \sum_{i=1}^{N} kB\$'_i \Delta h_i \tag{7-22}$$

The computations required to evaluate equations 7-19 through 7-22 are straightforward though they are time consuming for large well groups. Warriner and Banks (1977) provide a FORTRAN code to compute discharge and drawdowns for partially penetrating well groups within an arbitrarily shaped source boundary.

c. Wells in Sloping Aquifer. If the regional potentiometric surface has a significant slope, the effect of superimposing the initial regional gradient on the well drawdowns must be considered. For example, when pumping from floodplain locations, the existing piezometric gradient from upland areas to the river may be as great as those caused by pumping from the well. The significant parameters for confined flow to a single well are shown in figure 7-7. At a large distance from the well, the regional flow net would not be affected. All flow into the well would be contained within the stream lines separated by the dimension f . Thus by Darcy's law for one-dimensional flow

$$Q = -k \frac{dh_1}{dx} Bf \tag{7-23}$$

where

Q = discharge from well (L^3/T)

k = permeability (L/T)

h_1 = total head (L) for regional flow alone

x = coordinate selected to be parallel to initial regional flow (L)

B = aquifer thickness (L)

f = width of flow lines enclosing all flow to well (L)

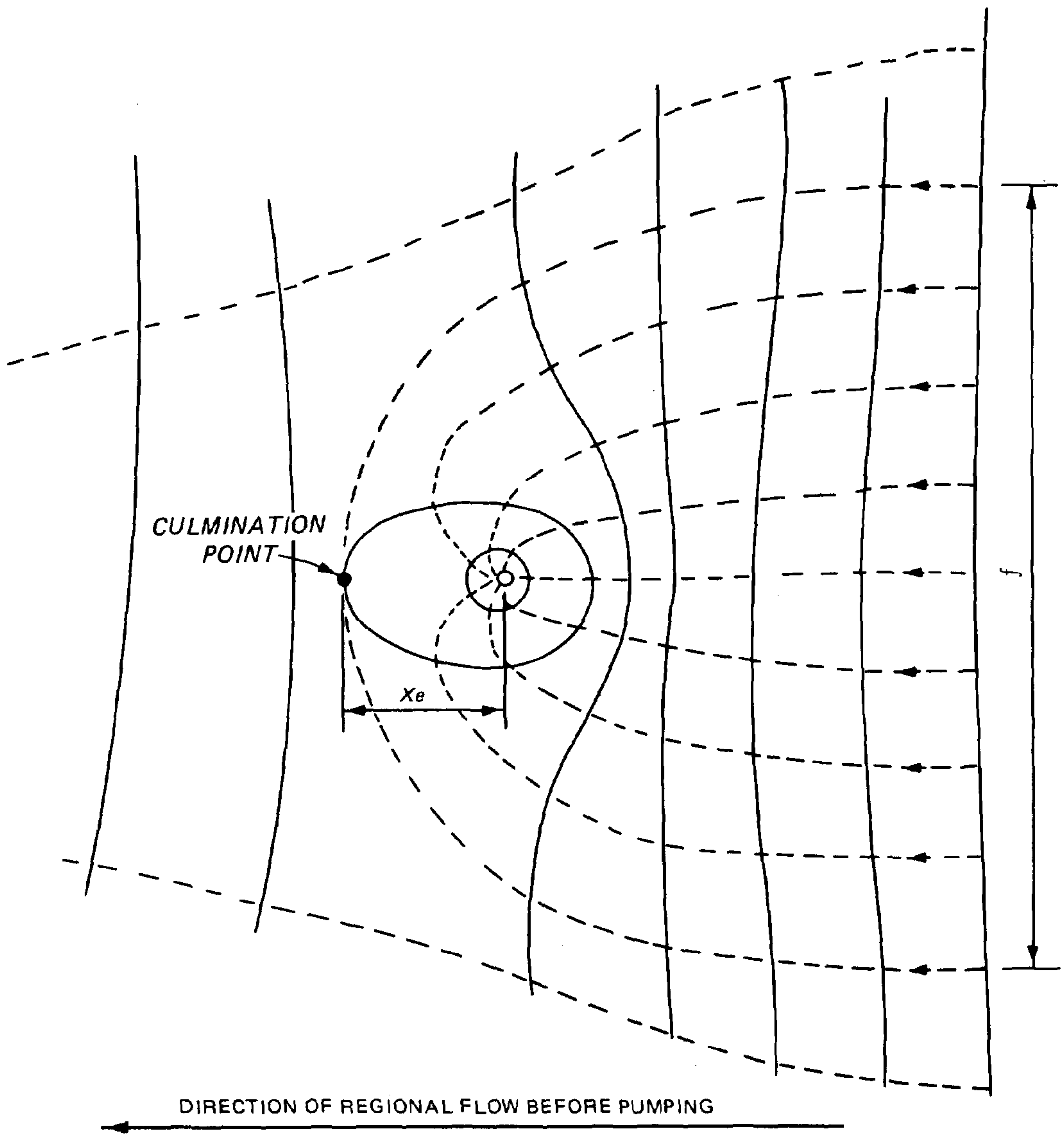


Figure 7-7. Superposition of well drawdown on regional gradient (courtesy of International Institute for Land Reclamation and Improvement[199])

The corresponding differential equation for the well would be

$$\frac{dh_2}{dr} = \frac{Q}{2\pi kB}\,\frac{1}{r} \tag{7-24}$$

where

h_2 = total head due to flow to well

$r = \sqrt{x^2 + y^2}$

At a distance X_e downgradient from the well, a groundwater divide develops (culmination point) at which

$$\frac{dh_2}{dr} = -\,\frac{dh_1}{dx} \tag{7-25}$$

In view of equations 7-23 and 7-24

$$\frac{Q}{2\pi kB}\,\frac{1}{X_e} = \frac{Q}{kBf} \tag{7-26}$$

or

$$X_e = \frac{f}{2\pi}$$

By substitution of equation 7-26 into equation 7-23

$$Q = 2\pi kX_eB\,\frac{dh_1}{dx} \tag{7-27}$$

By integrating equations 7-24 and 7-27

$$h_1 = \frac{Q}{2\pi kX_eB}\,X + c_1$$

(7-28)

$$h_2 = \frac{Q}{2\pi kB}\,\ln r + c_2$$

and superimposing the effects

$$h(x,y) = h_1 + h_2 = \frac{QX}{2\pi kBX_e} + \frac{Q}{4\pi kB} \ln (x^2 + y^2) + \text{constant} \tag{7-29}$$

The distance X_e can be removed from the expression by substitution of equation 7-27

$$h(x,y) = ix + \frac{Q}{4\pi kB} \ln (x^2 + y^2) + \text{constant} \tag{7-30}$$

where $i = \frac{dh_1}{dx}$ the regional slope of the aquifer.

For conditions of unconfined flow, the regional gradient would be defined by a parabola

$$h_1^2 = \frac{2Q}{Kf} x + \text{constant}$$

which when combined with the well equations for unconfined flow gives

$$h^2(x,y) = \frac{Q}{\pi kB} \left[\frac{x}{X_e} + \frac{1}{2} \ln (x^2 + y^2) \right] + \text{constant} \tag{7-31}$$

d. Layered Aquifers. Natural soils often occur in layers and a well may penetrate units having different permeabilities. If flow to the well is horizontal, the simple well equations can be used by assigning an average value of permeability given by

$$k_{avg} = \sum_{m=1}^{N} \frac{k_m d_m}{d} \tag{7-32}$$

where

k_m = horizontal permeability of layer m

d_m = thickness of layer m

d = total thickness of layers

Note that the permeability determined from a field pumping test is an average of all units penetrated by the pumping well. A case where vertical flow can be important is shown in figure 7-8. The discharging well is pumping from a

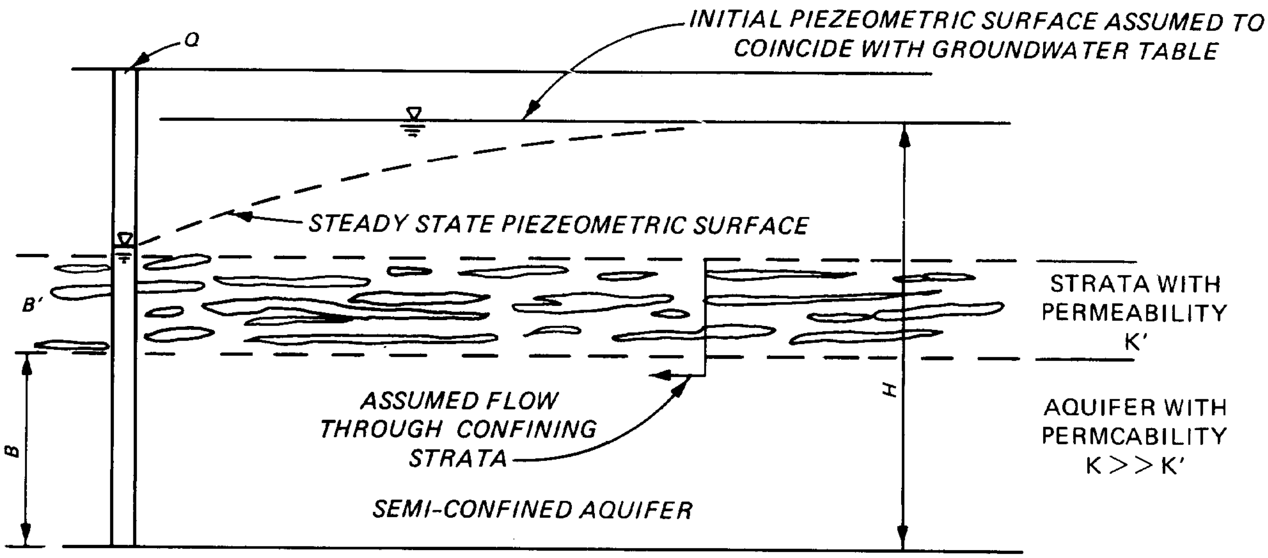


Figure 7-8. Flow to well with significant vertical flow through confining layer (courtesy of John Wiley and Sons[164])

permeable unit overlain by a less permeable unit through which significant vertical flow can occur. The flow to the well is given by

$$H - h = \frac{Q}{2\pi kB} K_o\left(\frac{r}{L}\right) \qquad (7\text{-}33)$$

where

H = original total head (L)

h = total head at distance r from well at steady state condition (L)

Q = discharge rate (L^3/T)

L = $\sqrt{kBC}$ (leakage factor) (L)

B = thickness of aquifer

C = B'/k' (L)

B' = thickness of overlying low permeability unit (L)

k' = permeability of overlying low permeability unit (L/T)

$k_o\left(\frac{r}{L}\right)$ = Hankel function (tabulated in table 7-1) (dimensionless)

7-5. Nonsteady State Flow. Nonsteady state flow may arise in several ways. When pumping is started, time is required to establish a virtually steady state condition. Flow during this period must be assumed to be nonsteady state. If pumping occurs intermittently, a steady state condition may not be established. Also, if large fluctuations occur at the source, potential steady state flow conditions are not maintained. The steady state condition can be viewed as the end condition that is reached after pumping for a long time period. In the design of a well system for seepage control, it is generally adequate to consider only the steady state condition. However, the determination of coefficient of permeability from test data often requires analysis based on nonsteady state condition. The duration of many well tests is too short to

Table 7-1. Values of K_o r/L for Selected Values of r/L to Evaluation Equation 7-33[a]

$\frac{r}{L}$ f	$f = 10^{-2}$	$f = 10^{-1}$	$f = 1.0$
1.0	4.721	2.427	0.421
2.0	4.028	1.753	0.114
3.0	3.623	1.372	0.035
4.0	3.336	1.114	0.011
5.0	3.114	0.924	0.004
6.0	2.933	0.777	0.000
7.0	2.780	0.660	0.000
8.0	2.647	0.565	0.000
9.0	2.531	0.487	0.000

Example: $\frac{r}{L} = 0.5$, $f = 10^{-1}$, $K(0.5) = 0.924$

(a) Prepared from more extensive tables presented by Kruseman and De Ridder (1970).

reliably establish the steady state condition. Also, in practice, hydrologic boundaries may be present within the steady state radius of influence. In either case the use of the steady state flow equations could lead to substantial error in determining the permeability.

a. Nonsteady State Confined Flow. Theis (1935) developed the following relationship for nonsteady state flow in a confined aquifer (Davis and DeWeist 1966):

$$H - h = \frac{Q}{4\pi kB} W(u) \tag{7-34}$$

where

Q = constant discharge rate (L^3/T)

k = permeability (L/T)

B = thickness of aquifer (L)

W(u) = function given in table 7-2

$$= -0.5772 - \ln u + u - \frac{u^2}{2 \times 2!} + \frac{u^3}{3 \times 3!} - \frac{u^4}{4 \times 4!} + \cdots$$

The parameter u is given by

$$u = \frac{r^2 S}{4kBt} \tag{7-35}$$

where

r = radius from well (L)

S = storage coefficient (dimensionless)

t = time from start of pumping (T)

The storage coefficient S represents the amount of water removed from storage as a result of consolidation of the aquifer and expansion of water in response to the decline in head. Physically S is given by

$$S = \rho g B(\alpha + n\beta) \tag{7-36}$$

Table 7-2. Values of W(u) for Selected Values of 1/u to Evaluate Equation 7-34[a]

$\frac{1}{f}\frac{1}{u}$	W(u) f = 1.0	W(u) f = 2.0	W(u) f = 8.0
10^{-1}	0.000	0.001	0.146
1	0.219	0.600	1.623
10	1.823	2.468	3.817
10^{2}	4.034	4.726	6.109
10^{3}	6.332	7.024	8.410
10^{4}	8.633	9.326	10.71
10^{5}	10.94	11.63	13.02
10^{6}	13.24	13.93	15.32
10^{7}	15.54	16.23	17.62
10^{8}	17.84	18.54	19.92
10^{9}	20.15	20.84	22.22
10^{10}	22.45	23.14	24.53
10^{11}	24.75	25.44	26.83
10^{12}	27.05	27.75	29.13
10^{13}	29.36	30.05	31.44
10^{14}	31.66	32.35	33.74

Example : $u = 0.005$, $\frac{1}{u} = 200$, $f = 2$, $W(u) = 4.726$

[a] Prepared by WES.

where

ρ = mass density fluid (m/L^3)

g = acceleration of gravity (L/T^2)

B = thickness of aquifer

α = bulk compressibility of aquifer (LT^2/M)

n = porosity (dimensionless)

β = bulk compressibility of fluid (LT^2/M)

The determination of the aquifer properties kb and S from equation 7-34 requires a complete drawdown versus time history for each observation piezometer. The Theis method for data analysis is based on the logarithmic representation of equations 7-34 and 7-35

$$\log (H - h) = \log[W(u)] + \log \left(\frac{Q}{4\pi kB}\right)$$

$$\log \left(\frac{r^2}{t}\right) = \log(u) + \log \left(\frac{S}{rkB}\right)$$

From the equations above it is seen that if Q is constant that log(H - h) varies with $\log(r^2/t)$ in the same way as log [W(u)] varies with log (u) regardless of the units used. Therefore, it should be possible to superimpose the data curve on the theoretical curve because the two curves are offset from each other only by the constant terms log $Q/4\pi kB$ and log S/4kB . By determining the value of the offsets from the superimposed curves, kb and S can be determined. The computation consists of the following steps:

(1) A plot is made of W(u) (log scale) versus u (log scale). This plot is referred to as the type curve.

(2) For each observation well, a plot is made of drawdown H - h (log scale) versus r^2/t (log scale).

(3) Superimpose the test data over the type curve in such a way that the drawdown data best fit the type curve (figure 7-9). The coordinate axes of the two curves should be kept parallel.

(4) Determine the values W(u) , u , H - h , and r^2/t from an arbitrarily chosen matching point on the two curves.

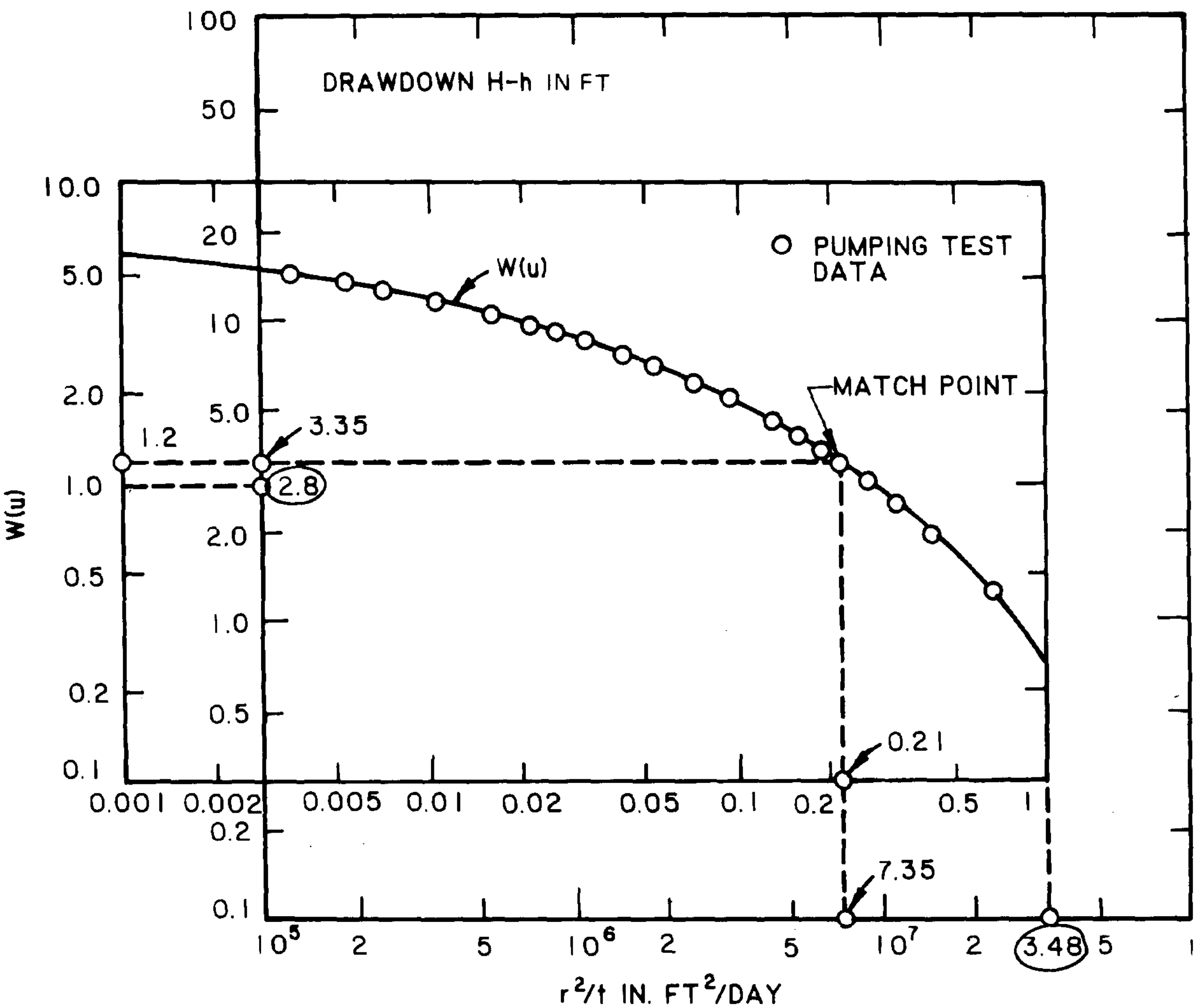


Figure 7-9. Use of type curve for analysis of nonsteady state flow (courtesy of John Wiley and Sons[164])

(5) Compute the value of kB from equation 7-34 using the matching point value of H - h and W(u) . Compute the value of S from equation 7-35 using the matching point values of u and r^2/t combined with the previously computed value of kB . The above procedure is carried out for each observation well. Ideally, the computed values of kB and S should be the same for all observation wells. Differences in the computed values may be caused by geologic variations in the aquifer and hydrologic boundaries not accounted for in the analysis.

b. Simple Method for Coefficient Determination (Jacob's Method). Jacob (1950) introduced a simplification to the determination of kB and S by noting that for small values of u (small r and/or large t) equation 7-34 reduces to (Davis and DeWeist 1966)

$$H - h = \frac{Q}{4\pi kB}\left(\ln\frac{1}{u} - \ln 1.78\right) ; \quad u \le 0.01 \tag{7-37}$$

Equation 7-37 can be written in a form convenient for graphical solution by substituting equation 7-35 and writing in terms of base 10 logarithms:

$$H - h = \frac{2.30Q}{4\pi kB}\log\frac{2.25T}{r^2S} + \frac{2.30Q}{4\pi kB}\log t \tag{7-38}$$

From equation 7-38 it is seen that the relationship between drawdown H - h and time t for a particular observation piezometer (r = constant) can be represented as a straight line on a plot of H - h versus log t (figure 7-10). The slope of the line is equal to $\frac{2.30Q}{4\pi t}$ Also, the time, t_o , corresponding to H - h = o gives

$$\frac{2.25^{Tt}o}{r^2S} = 1$$

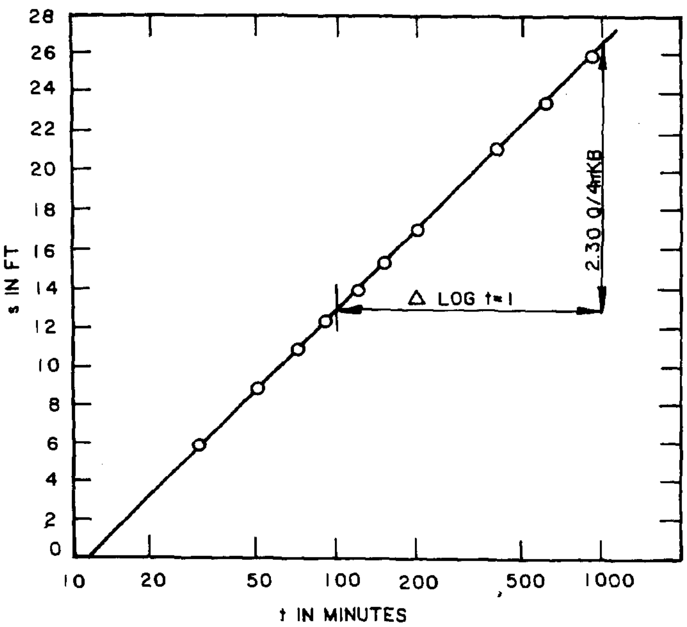
which can be used to determine S . An alternative analysis consists of plotting H - h versus log r . The following relationship can be obtained by rearranging the term in equation 7-38.

$$H - h = \frac{2.30Q}{4\pi kB}\log\frac{2.25kBt}{S} - \frac{2.30Q}{2\pi kB}\log r \tag{7-39}$$

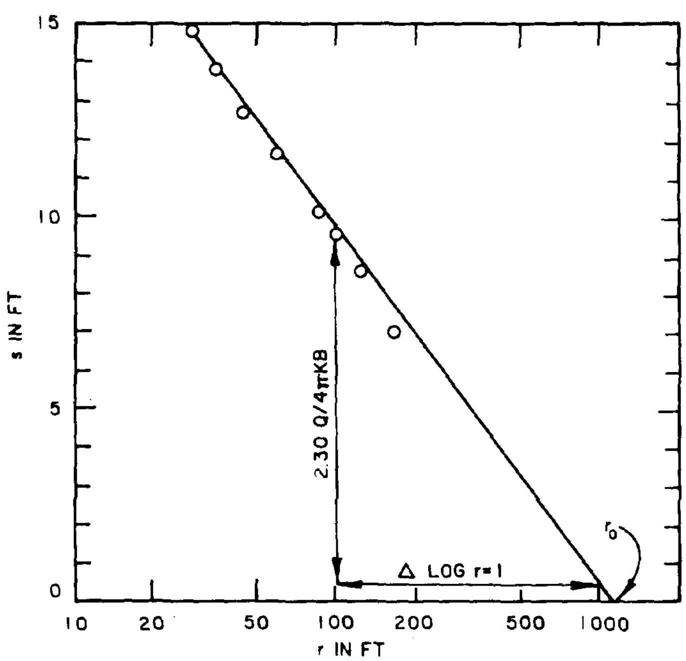
Equation 7-39 defines a straight line on a plot of H - h versus r (figure 7-10b). The slope of the line is - $\frac{2.30Q}{2\pi kB}$ and can be used to determine kb . The line intersects the H - h = o axis at r_o . This intercept can be used to determine S from

$$\frac{2.25kBt}{r_o^2S} = 1$$

Note that r_o represents the radius of influence for the well at time equals t . Thus the radius of influence for the steady state condition r_e is equal to r_o as t tends to infinity. This implies that the radius of influence



a. One observation well



b. Simultaneous observations

Figure 7-10. Use of Jacob approximation for nonsteady state flow (courtesy of John Wiley and Sons[164]) expands indefinitely and cannot be defined. However, the value of r_e selected has a relatively small influence on computed drawdowns near the well and equation 7-39 can be used to determine reasonable values for r_e .

c. Nonsteady Unconfined Flow with Vertical Gravity Drainage (Delayed Yield). Initial response (generally after first few minutes of pumping) is given by (Kruseman and DeRidder 1970)

$$H - h = \frac{Q}{4\pi kB} W(u_A \ , \ r/B) \tag{7-40}$$

where

$$u_A = \frac{r^2 S_A}{4kBt}$$

S_A = storage coefficient for instantaneous release of water from storage

$W(u_A \ , \ r/B)$ = Boulton well function (figure 7-11a)

r/B = formation constant to be determined from piping test data

Later time response is given by

$$H - h = \frac{Q}{4\pi kB} W(u_y \ , \ r/B) \tag{7-41}$$

where

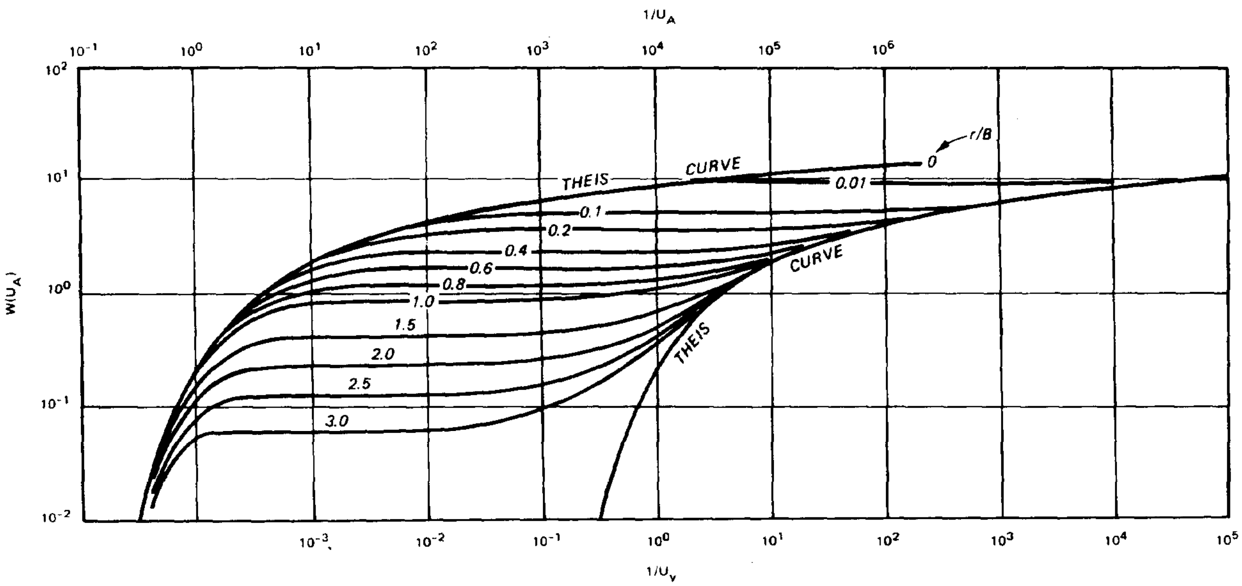
$$u_y = \frac{r^2 S_y}{4kBt}$$

S_y = specific yield

$W(u_y \ , \ r/B)$ = delayed yield well function

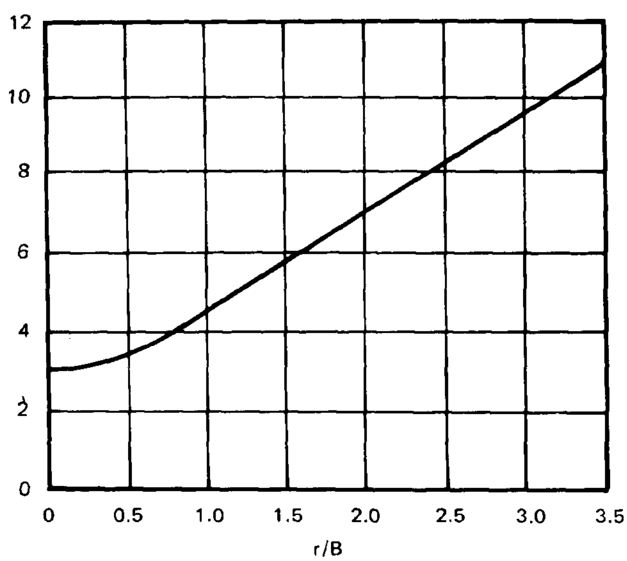
The application of equations 7-40 and 7-41 through use of a type-curve is similar to that of equation 7-34. The following should also be noted:

(1) Type curves for several values of r/B should be plotted. The curve giving the best fit to the initial time-drawdown data is used to estimate r/B .



a FAMILY OF BOULTON TYPE CURVES: W (U_A, r/B) VERSUS $1/u_A$ AND W (U_y, r/b) VERSUS $1/U_y$ FOR DIFFERENT VALUES OF r/B.

a. Family of Boulton type curves: $W(U_A$, r/B) versus $1/u_A$ and $W(U_y/r/b)$ versus $1/U_y$ for different values of r/B



b. Boulton's delay index curve

Figure 7-11. Type curves for Boulton's analysis of nonsteady unconfined flow with delayed yield (courtesy of International Institute for Land Reclamation and Improvement[199])

(2) The time-drawdown data overlay may be moved to obtain the best fit for the latter time-drawdown data. Both initial time and latter time fits should give the same value of r/B and kB .

(3) Eventually, the effects of vertical gravity drainage become negligible and the latter time curve merges with the Theis curve. The time-coordinate where the two curves merge is determined from Boulton's delay-index curve (figure 7-11b).

(4) A number of type-curve solutions to the problem of nonsteady unconfined flow to wells have been developed (Fetter 1980). For example, Neuman (1975) presented a type-curve method similar to Boulton's that accounts for anisotropy of the aquifer.

d. Nonsteady Confined Flow with Vertical Drainage Through Confining Layer (Leaky Aquifer). The leaky aquifer equation for nonsteady flow is based on the assumptions that flow to the well is horizontal and vertical flow is restricted to seepage through the confining layer. These assumptions are identical to those made for the steady state case described by equation 7-33. The drawdown is given by

$$H - h = \frac{Q}{4\pi kB} W\left(u \ , \ \frac{r}{L}\right) \tag{7-42}$$

where

$u = \dfrac{r^2 S}{4kBt}$

r = radius from well (L)

S = storage coefficient (dimensionless)

k = permeability (L/T)

t = time from start of pumping (T)

L = leakage factor (L) = $\dfrac{k\ BB'}{k'}$

B = thickness of aquifer (L)

B' = thickness of confining unit (L)

k' = permeability of confining unit (L/T)

$W(u\ ,\ r/L)$ = well function given in figure 7-12

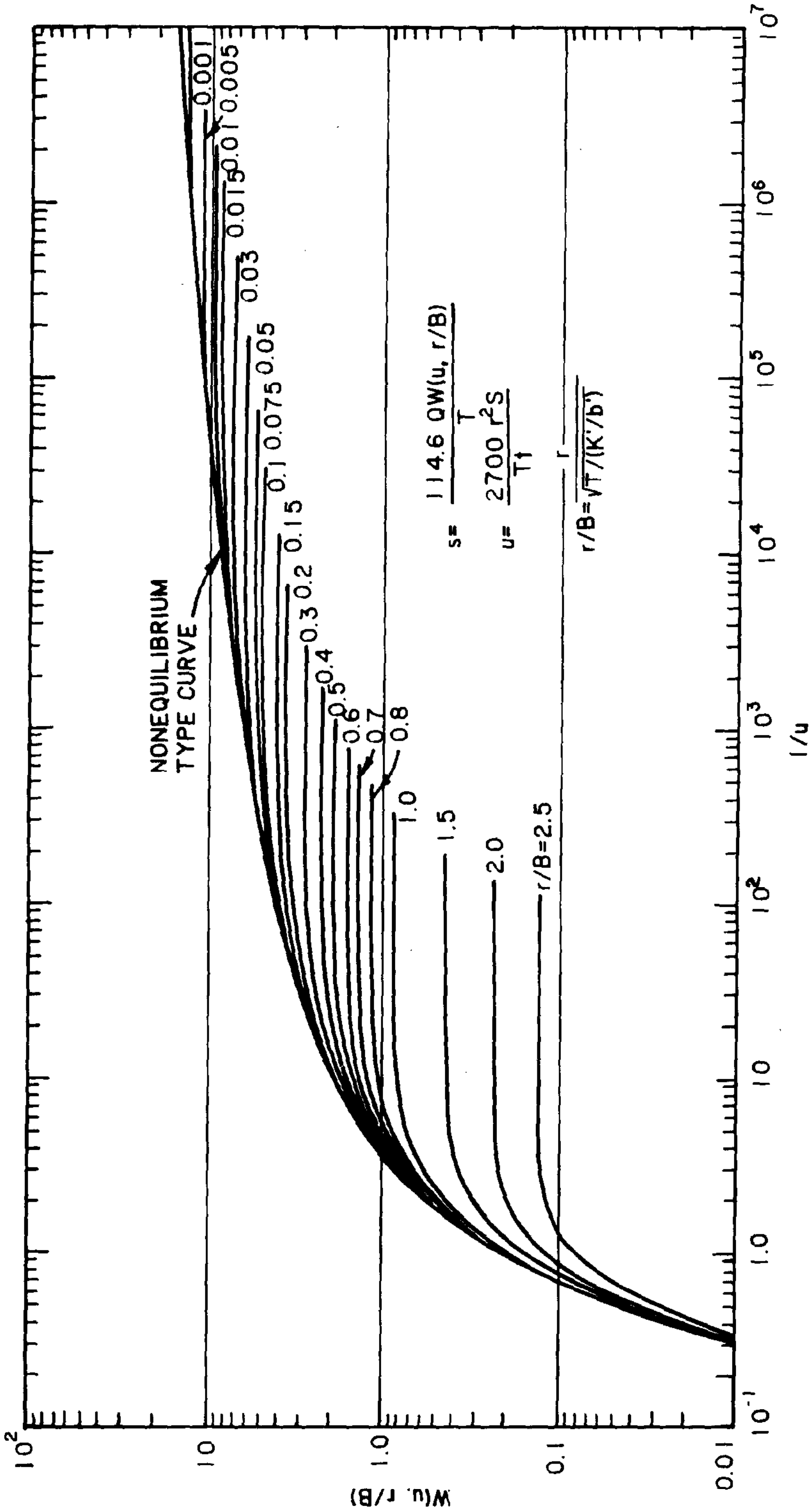


Figure 7-12. Type curve for nonsteady flow in a semiconfined aquifer (courtesy of John Wiley and Sons[164])

The application of the type curve method for the leaky aquifer problem is similar to the application to the delayed yield problem. The time-drawdown data are matched to the standard type curve with the curve giving the best fit being used to estimate r/B .

e. Nonsteady Unconfined Flow with Little Vertical Drainage. If the delayed response component of the drawdown is small, the Theis equation (equation 7-34) can be used to analyze the flow by inserting a "corrected" drawdown into the flow equation. The corrected drawdown is given by

$$(H - h)' = (H - h) - \left[\frac{(h - H)^2}{2H}\right] \qquad (7\text{-}43)$$

f. Nonsteady Flow with Hydrologic Boundaries. The method of superposition presented for steady-state flow problems (equation 7-9) is applicable to nonsteady flow problems. Therefore, the image well method can be used to investigate the effects of hydrologic boundaries. For example, the image well analysis for a discharging well near a river (recharge boundary) is (Davis and Dewiest 1966)

$$H - h = \frac{Q}{4\pi kB}\left[W(u_R) - W(u_I)\right] \qquad (7\text{-}44)$$

where

$u_R = \dfrac{r_R^2 S}{4kBt}$

$u_I = \dfrac{r_I^2 S}{4kBt}$

r_R = radius from real well (L)

r_I = radius from image well (L)

k = coefficient of permeability (L/T)

S = storage coefficient (dimensionless)

B = aquifer thickness (L)

t = time from start of pumping (T)

Note, then, when the function W(u) can be replaced with a logarithmic approximation, as in the Jacob's method (equation 7-37), equation 7-44 can be approximated as

$$H - h = \frac{Q}{4\pi kB} \ell n \frac{r_R^2 S}{4kBt} - \ell n \frac{r_I^2 S}{4kBt} = \frac{Q}{2\pi kB} \ell n \frac{r_I}{r_R} \qquad (7\text{-}45)$$

From equation 7-45 it is seen that as u becomes small, flow becomes virtually steady state (compare equation 7-45 with the steady state case, equation 7-11). Thus the presence of a recharge boundary in an aquifer tends to shorten the time needed to reach steady state (Davis and Dewiest 1966).